Generalized Magneto-Thermoelastic Medium with Variable Material Properties Subjected to Ramp-Type Heating

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ABSTRACT: The equations of magneto-generalized thermoelasticity with one relaxation time with variable electrical and thermal conductivity for one-dimensional problems are cast into a matrix form using the state–space approach and Laplace transform techniques. The resulting formulation is applied to a problem of a half space where the bounded plane is subjected to a ramp-type heating and a traction free. This takes place when a constant magnetic field permeates the medium in the absence of an external electric field. The inversion of the Laplace transform is carried out using a numerical approach. Numerical results for the temperature, the displacement, and the stress distributions are given and illustrated graphically. © 2011 Wiley Periodicals, Inc. J Appl Polym Sci 124: 5209–5219, 2012

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INTRODUCTION

For the last three decades, serious attention has been paid to the generalized thermoelasticity theories in solving thermoelastic problems in place of the classical uncoupled/coupled theory of thermoelasticity. The absence of any elasticity term in the heat conduction equation for uncoupled thermoelasticity appears to be unrealistic; due to the mechanical loading of an elastic body, the produced strain causes variation in the temperature field. Moreover, the parabolic type of the heat conduction equation results in an infinite velocity of the propagating thermal wave which also contradicts the actual physical phenomena. Introducing the strain-rate term in the uncoupled heat conduction equation, Biot[1] extended the analysis to incorporate coupled thermoelasticity. Although the first shortcoming was over, there remained the parabolic type partial differential equation of the heat conduction, which leads to the paradox of the infinite velocity of the thermal wave. To eliminate this paradox, generalized thermoelasticity theory has been developed subsequently. The development of this theory was accelerated by the advent of the second sound effects observed experimentally by Ackerman[2,3] in materials at a very low temperature. In heat transfer problems involving very short-time intervals and/or very high heat fluxes, it has been revealed that the inclusion of the second sound effects to the original theory yields results that are realistic and very much different from those obtained with classical theory of elasticity.

Due to an advantages of pulsed lasers, fast burst nuclear reactors and particle accelerators, which can supply heat pulses with a very fast time rise[4,5]; generalized thermoelasticity theory is receiving serious attention from many researchers. The development of the second sound effect has been nicely reviewed by Chandrasekhariah.[6] Currently, two different models of generalized thermoelasticity are being extensively used, one proposed by Lord and Shulman[7] and the other proposed by Green and Lindsay.[8] The L–S theory suggests one relaxation time and according to this theory, only Fourier's heat conduction equation is modified; on the other hand, G–L theory suggests two relaxation times, and both the energy equation and the motion equation are modified. Contrary to the L–S theory, the G–L theory does not violate Fourier's law of heat conduction when the solid has a center of symmetry.

Bahar and Hetnarski[9–11] developed a method for solving coupled thermoelastic problems by using the state–space approach in which the problem is rewritten in terms of the state–space variables, namely the temperature and the displacement gradients. Erbay and Suhubi[12] studied the longitudinal wave propagation in an infinite circular cylinder, which is assumed to be made of the generalized thermoelastic material, and thereby obtained the dispersion

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relation when the surface temperature of the cylinder was kept constant. Generalized thermoelasticity problems for an infinite body with a circular cylindrical hole and for an infinite solid cylinder were solved, no need for this word by Furukawa et al.[13,14] A problem of generalized thermoelasticity was solved by Sherief,[15] by adopting the state–space approach. Youssef and El-Bary[26] used the same approach to solve the problem of generalized thermoelastic infinite layer subjected to ramp-type thermal and mechanical loading under three theories. Chandrasekharaiah and Murthy[16] studied the thermoelastic interactions in an isotropic homogeneous unbounded linear thermoelastic body with a spherical cavity, in which the field equations were taken in unified forms covering the coupled, L–S, and G–L models of thermoelasticity. The effects of the mechanical and thermal relaxations in a heated viscoelastic medium containing a cylindrical hole were studied by Misra et al.[17] Investigations concerning interactions between magnetic and thermal fields in deformable bodies were carried out by Maugin[18] as well as by Eringen and Maugin.[19] Subsequently Abd-Alla and Maugin[20] conducted a generalized theoretical study by considering the mechanical, thermal, and magnetic fields in centro-symmetric magnetizable elastic solids.

Among the contributions in the context of the theory of L–S, El-Maghraby and Youssef[21] used the state–space approach to solve a thermomechanical shock problem. Sherief and Youssef[22] were able to achieve the short-time solution for a problem in magneto-thermoelasticity. Youssef[23] constructed a model representing the modulus dependability of elasticity and thermal conductivity on the reference temperature and solved a problem of an infinite material with a spherical cavity

Almost, all the mechanical or thermal loading on the bounding surface is considered to be in the form of a shock. However, the sudden jump of the load is merely an idealized situation because it is impossible to realize a pulse described mathematically by a step function; even very rapid rise time (of the order of 10^{-9} s) may be slow in terms of the continuum. This is particularly true in the case of second sound effects when the thermal relaxation times for typical metals are less than 10^{-9} s. Thus, a finite rise time of an external load (mechanical or thermal) applied on the surface should be considered while studying a practical problem. Considering this aspect of rise of time, Misra et al.[24] solved some problems subjected to a ramp-type heating at the bounding surface. In Ref.[25] Ezzt et al. used the normal mode analysis to solve the equations of the theory of electro-, magneto-, and thermoviscoelasticity, and the results were applied to a problem of a rotating thick plate subject to heat on parts of the upper and lower surfaces of the plate that varies exponentially with time.

The present investigation is devoted to a study of the induced temperature and stress fields in an elastic half space under the purview of generalized thermoelasticity in a unified system of field equations. The infinite half-space continuum is considered to be made of an isotropic homogeneous thermoelastic material; the bounding plane surface being subjected to a ramp-type heating and traction free with constant magnetic field permeates the medium normal to the bounding plane in the absence of an external electric field. The rationale behind the study of this type of heating is that the temperature of the bounding surface cannot be elevated instantaneously. Thus, a finite time of rise of temperature is required for this purpose and to show the effect of the magnetic field on the behavior of the metal. By adopting the state–space approach,[15] an exact solution for the problem is first obtained in Laplace transform space. Because the response is of more interest in the transient state, the inversions have been carried out numerically. The derived expressions are computed numerically for copper, and the results are presented in a graphical form.

The governing equations

A homogenous isotropic thermoelastic conducting solid occupying the region $0 \leq x < \infty$, whose state depends only on the space variables "x" and the time "t" and for which the displacement vector has component ($u(x,t)$,0,0) will be considered. A constant magnetic field with component (0, $H_{o,}$ 0) permeates the medium in the absence of an external electric field.

The heat equation

$$\left(K\,\theta_{,i}\right)_{,i} = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left[\frac{K}{\kappa}\,\dot{\theta} + \gamma\, T_o\, \dot{e}\right], \quad i = 1,\ 2,\ 3 \tag{1}$$

We will consider the thermal conductivity to be variable in the form[27]:

$$K = K\,(\theta) = K_o(1 + K_1\theta) \tag{2}$$

where K_1 is a negative small constant, κ (diffusivity) is constants, and K_o is the thermal conductivity when it is independent on temperature ($K = K_o$ when $K_1 = 0$ or $T = T_o$).

Applying the mapping[28] illustrated in

$$\vartheta = \frac{1}{K_o}\int_0^{\theta} K\,(\theta')d\theta' \tag{3}$$

By using eq. (2) into the mapping in (3), one can obtain

$$\vartheta = \theta + \frac{K_1}{2}\theta^2 \quad (4)$$

And relatively the heat equation in a one-dimensional problem will take the form

$$\frac{\partial^2\vartheta}{\partial x^2} = \left[\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right]\left[\frac{\vartheta}{\kappa} + \frac{\gamma\, T_o}{K_o}\frac{\partial u}{\partial x}\right] \quad (5)$$

The equations of motion presented in Reference 27

$$\rho\ddot{u}_i = \left(\vec{J}\wedge\vec{B}\right)_i + \sigma_{ij,j} \quad (6)$$

where $\vec{B}$ is the magnetic induction vector given by

$$\vec{B} = \mu_o\vec{H} \quad (7)$$

and $\vec{J}$ is the conduction current density given by Ohm's law

$$\vec{J} = \sigma\left[\vec{E} + \frac{\partial\vec{u}}{\partial t}\wedge\vec{B}\right] \quad (8)$$

where $\vec{E}$ is the electrical intensity vector.

The constitutive equations have the form[27]:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma\theta)\delta_{ij} \quad (9)$$

where $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$.

The component of the electromagnetic induction vector are given by $B_x = B_z = 0$, $B_z = B_o = \mu_o H_o$ (Constant), while the component of $\vec{F} = \vec{J}\wedge\vec{B}$ is given as (neglecting the nonlinear term),

$$F_x = -\sigma B_o^2\frac{\partial u}{\partial t}, \qquad F_y = F_z = 0 \quad (10)$$

The Wiedemann–Franz law[29] states that for metals at not too low temperature, the ratio of the thermal conductivity K to the electrical conductivity σ is directly proportional of particular metal

$$\frac{K}{\sigma} = LT \quad (11)$$

where $L = 2.45 \times 10^{-8}$ [W Ω K^{-2}] is the Lorenz number.[27]

Using eq. (2), one can obtain

$$\sigma = \frac{K_o}{LT_o}(1 + K_1\theta)$$

For linearity, the following approximation can be made

$$\sigma \cong \frac{K_o}{LT_o}(1 + KT_o) \quad (12)$$

Using eqs. (6), (9), (10), and (12), the equation of motion can be obtained as:

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \frac{\gamma}{(1 + K_1T_o)}\frac{\partial\vartheta}{\partial x} - \frac{H_o^2\,\mu_o^2K_o(1 + K_1T_o)}{L\,T_o}\frac{\partial u}{\partial t} = \rho\frac{\partial^2 u}{\partial t^2} \quad (13)$$

where $\frac{\partial\theta}{\partial x} = \frac{1}{(1+K_1\theta)}\frac{\partial\vartheta}{\partial x} \cong \frac{1}{(1+K_1T_o)}\frac{\partial\vartheta}{\partial x}$ and the constitutive equation in the form

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \frac{\gamma}{(1 + K_1T_o)}\vartheta \quad (14)$$

The nondimensional variables are introduced as

$$x' = \frac{c_o}{\kappa}x,\ u' = \frac{c_o}{\kappa}u,\ t' = \frac{c_o^2}{\kappa}t,\ \vartheta' = \frac{\vartheta}{T_o},\ \tau_o' = \frac{c_o^2}{\kappa}\tau_o,\ \sigma' = \frac{\sigma}{\lambda + 2\mu},$$

where $c_o^2 = \frac{\lambda+2\mu}{\rho}$.

Hence,

$$\frac{\partial^2\vartheta}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\left(\vartheta + \varepsilon\frac{\partial u}{\partial x}\right) \quad (15)$$

$$\frac{\partial^2 u}{\partial x^2} - a\frac{\partial\vartheta}{\partial x} - M\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} \quad (16)$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - a\,\vartheta \quad (17)$$

where $a = \frac{\gamma\,T_o}{(\lambda+2\mu)\,(1+K_1T_o)}$, $M = \frac{C_E\,H_o^2\,\mu_o^2\,\kappa^2\,(1+K_1T_o)}{c_o^2\,L\,T_o}$, and $\varepsilon = \frac{\gamma}{\rho\,C_E}$.

In the above equations, the primes were dropped for convenience.

Applying the Laplace transform defined by the formula

$$\bar{f}(s) = \mathrm{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,dt$$

Yield

$$\frac{\partial^2\bar{\vartheta}}{\partial x^2} = \left(s + \tau_o s^2\right)\left(\bar{\vartheta} + \varepsilon\frac{\partial\bar{u}}{\partial x}\right) \quad (18)$$

$$\frac{\partial^2\bar{u}}{\partial x^2} = \left(s\,M + s^2\right)\bar{u} + a\frac{\partial\bar{\vartheta}}{\partial x} \quad (19)$$

$$\bar{\sigma}_{xx} = \frac{\partial\bar{u}}{\partial x} - a\,\bar{\vartheta} \quad (20)$$

where

$$u(x,t)|_{t=0} = \vartheta(x,t)|_{t=0} = 0$$

Choosing as state variable the temperature increment, the displacement component in the x-direction, and their gradient, then eqs. (18) and (19) can be written in matrix form as

$$\frac{d\,\bar{V}(x,s)}{d\,x} = A(s)\,\bar{V}(x,s) \tag{21}$$

where

$$\bar{V}(x,s) = \begin{bmatrix} \bar{u}(x,s) \\ \bar{\vartheta}(x,s) \\ \bar{u}'(x,s) \\ \bar{\vartheta}'(x,s) \end{bmatrix}$$

and

$$A(s) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ h^2 & 0 & 0 & a \\ 0 & g^2 & \varepsilon\,g^2 & 0 \end{bmatrix} \tag{22}$$

Having $h^2 = sM + s^2$ and $g^2 = s + \tau_o s^2$.

The formal solution of system (21) can be written in the form

$$\bar{V}(x,s) = \exp[A(s)\,x]\,\bar{V}(0,s) \tag{23}$$

Thus the well-known Cayley–Hamilton theorem is used to find the form of the matrix exp $(A(s)\,x)$.

The characteristic equation of the matrix $A(s)$ can be written as

$$k^4 - \lfloor a\,\varepsilon\,g^2 + g^2 + h^2 \rfloor\;k^2 + g^2\,h^2 = 0 \tag{24}$$

the roots of this equation, namely k_1^2 and k_2^2, satisfy the following relations:

$$k_1^2 + k_2^2 = a\,\varepsilon\,g^2 + g^2 + h^2 \tag{25}$$

$$k_1^2 k_2^2 = g^2 h^2 \tag{26}$$

The Taylor series expansion of the matrix exponential has the form

$$\exp[A(s)\,x] = \sum_{n=0}^{\infty} \frac{[A(s)\,x]^{\,n}}{n!} \tag{27}$$

Using Cayley–Hamilton theorem again, both A^4 and the higher orders of the matrix A can be expressed in terms of I, A, A^2, and A^3, where I is the unit matrix of the fourth order.

Thus, the infinite series in eq. (27) can be reduced to

$$\exp[A(s)\,x] = L(x,s) = a_o I + a_1 A + a_2 A^2 + a_3 A^3 \tag{28}$$

where a_o–a_3 are coefficients that depend on x and s. Based on Cayley–Hamilton theorem, the characteristic roots $\pm k_1$ and $\pm k_2$ of matrix A must satisfy eq. (28), thus

$$\begin{aligned} \exp(k_1\,x) &= a_o + a_1\,k_1 + a_2\,k_1^2 + a_3\,k_1^3 \\ \exp(-k_1\,x) &= a_o - a_1\,k_1 + a_2\,k_1^2 - a_3\,k_1^3 \\ \exp(k_2\,x) &= a_o + a_1\,k_2 + a_2\,k_2^2 + a_3\,k_2^3, \\ \exp(-k_2\,x) &= a_o - a_1\,k_2 + a_2\,k_2^2 - a_3\,k_2^3 \end{aligned} \tag{29}$$

The solution of this system is given by

$$\begin{aligned} a_o &= \frac{k_1^2\cosh(k_2x) - k_2^2\cosh(k_1x)}{k_1^2 - k_2^2} \\ a_1 &= \frac{\frac{k_1^2}{k_2}\sinh(k_2x) - \frac{k_2^2}{k_1}\sinh(k_1x)}{k_1^2 - k_2^2}, \\ a_2 &= \frac{\cosh(k_1x) - \cosh(k_2x)}{k_1^2 - k_2^2} \\ a_3 &= \frac{k_2\,\sinh(k_2x) - k_1\,\sinh(k_1x)}{k_2\,k_1\,(k_1^2 - k_2^2)} \end{aligned} \tag{30}$$

Substituting the expressions defined in eq. (30) into eq. (28) and computing A^2 and A^3 yield,

$$\exp[A(s)\,x] = L(x,s) = \lfloor \ell_{i\,j}(x,s) \rfloor \qquad i,j = 1,\,2,\,3,\,4, \tag{31}$$

where the components $\ell_{ij}(x,s)$ are given by

$$\begin{aligned} \ell_{11} &= \frac{1}{k_1^2 - k_2^2}\left[(h^2 - k_2^2)\cosh(k_1\,x) - (h^2 - k_1^2)\cosh(k_2\,x)\right] \\ \ell_{12} &= \frac{ag^2}{k_1^2 - k_2^2}\left[\frac{\sinh(k_1\,x)}{k_1} - \frac{\sinh(k_2\,x)}{k_2}\right] \\ \ell_{13} &= \frac{1}{k_1^2 - k_2^2}\left[\frac{(k_1^2 - g^2)\sinh(k_1\,x)}{k_1} - \frac{(k_2^2 - g^2)\sinh(k_2\,x)}{k_2}\right] \\ \ell_{14} &= \frac{a}{k_1^2 - k_2^2}\,[\cosh(k_1\,x) - \cosh(k_2\,x)] \\ \ell_{21} &= \frac{\varepsilon\,g^2\,h^2}{k_1^2 - k_2^2}\left[\frac{\sinh(k_1\,x)}{k_1} - \frac{\sinh(k_2\,x)}{k_2}\right] \\ \ell_{22} &= \frac{1}{k_1^2 - k_2^2}\left[(g^2 - k_2^2)\cosh(k_1\,x) - (g^2 - k_1^2)\cosh(k_2\,x)\right] \\ \ell_{23} &= \frac{\varepsilon\,g^2}{k_1^2 - k_2^2}\,[\cosh(k_1\,x) - \cosh(k_2\,x)], \\ \ell_{24} &= \frac{1}{k_1^2 - k_2^2}\left[\frac{(k_1^2 - h^2)\sinh(k_1\,x)}{k_1} - \frac{(k_2^2 - h^2)\sinh(k_2\,x)}{k_2}\right], \\ \ell_{31} &= h^2\,\ell_{13}, \\ \ell_{32} &= \frac{a}{\varepsilon}\ell_{23}, \\ \ell_{33} &= \frac{1}{k_1^2 - k_2^2}\left[(k_1^2 - g^2)\cosh(k_1\,x) - (k_2^2 - g^2)\cosh(k_2\,x)\right], \\ \ell_{34} &= \frac{a}{k_1^2 - k_2^2}\,[k_1\sinh(k_1\,x) - k_2\sinh(k_2\,x)], \end{aligned} \tag{32}$$

$$\ell_{41} = \frac{\varepsilon\, g^2\, h^2}{a}\, \ell_{14},$$

$$\ell_{42} = g^2 \ell_{24},$$

$$\ell_{43} = \frac{\varepsilon\, g^2}{a}\, \ell_{34},$$

$$\ell_{44} = \frac{1}{k_1^2 - k_2^2}\left[\left(k_1^2 - h\right)\cosh(k_1\, x) - \left(k_2^2 - h^2\right)\cosh(k_2\, x)\right],$$

Application

A half-space homogeneous elastic medium occupying the region $0 \leq x < \infty$ with quiescent initial state having $\sigma(x,t)\,|_{x=\infty} = \theta(x,t)\,|_{x=\infty} = u(x,t)\,|_{x=\infty} = 0$ is considered.

The bounding plane $x = 0$ is subjected to a ramp-type heating that takes the form

$$\theta(0,t) = \begin{bmatrix} 0 & \text{for } t \leq 0 \\ \theta_1 \frac{t}{t_o} & \text{for } 0 < t \leq t_o \\ \theta_1 & \text{for } t > t_o \end{bmatrix} \tag{33}$$

and is considered to be traction free

$$\sigma_{x\,x}(0,t) = 0 \tag{34}$$

where θ_1 is a constant, and t_o is the ramping parameter.

Since the intention for the solution is to vanished at infinity, then the positive exponentials in eqs. (32) should be rejected. This is done by replacing each $\cosh(k_i x)$ by $\frac{1}{2}\exp(-k_i\, x)$ and each sin $h(k_i x)$ by $-\frac{1}{2}\exp(-k_i\, x)$, $i = 1, 2$ in eqs. (32).

We now apply the state–space approach described previously to this problem. The two components of the transformed initial state $\overline{V}(0,s)$ are known as,

$$\bar{\vartheta}(0,s) = \delta(s), \tag{35}$$

where $\delta(s) = \left[\frac{-e^{-s\,t}[K_1(s\,t_o+1)\,+s\,t_o]+K_1+s\,t_o}{s^3\,t_o^2}\right]\theta_1$, which follows from eqs. (4) and (35), and

$$\bar{u}'(0,s) = a\,\delta \tag{36}$$

which follows from eqs. (20), (34), and (35).

To obtain the two remaining components $\bar{u}(0,s)$ and $\bar{\vartheta}'(0,s)$, $x = 0$ should be substituted in both sides of eq. (23), and the necessary matrix operation should be performed to obtain a system of linear algebraic equations for the unknowns $\bar{u}(0,s)$ and $\bar{\vartheta}'(0,s)$, whose solution gives

$$\bar{u}(0,s) = -\frac{a\,\delta}{(k_1 + k_2)}, \tag{37}$$

$$\bar{\vartheta}'(0,s) = -\frac{\delta\left[k_1^2 + k_1 k_2 + k_2^2 - h^2\right]}{(k_1 + k_2)}. \tag{38}$$

Substituting eqs. (37) and (38) in the right-hand side of eq. (23) and using eqs. (31) and (32) provide

$$\bar{u}(x,s) = -\frac{a\,\delta}{\left(k_1^2 - k_2^2\right)}\left[k_1\, e^{-k_1\,x} - k_2\, e^{-k_2\,x}\right], \tag{39}$$

$$\bar{\vartheta}(x,s) = \frac{\delta}{\left(k_1^2 - k_2^2\right)}\left[\left(k_1^2 - h^2\right) e^{-k_1\,x} - \left(k_2^2 - h^2\right) e^{-k_2\,x}\right], \tag{40}$$

where the temperature increment $\bar{\theta}(x,s)$ can be obtained by solving eq. (4) to give

$$\bar{\theta}(x,s) = \frac{-1 + \sqrt{1 + 2\,K_1\,\vartheta(x,s)}}{K_1}, \tag{41}$$

where $\lim_{k_1 \to 0} \bar{\theta}(x,s) = \bar{\vartheta}(x,s)$

Using eqs. (20), (39), and (40) result in

$$\bar{\sigma}_{xx}\,(x,s) = \frac{a\,\delta\, s^2}{\left(k_1^2 - k_2^2\right)}\left[\, e^{-k_1\,x} - \, e^{-k_2\,x}\right] \tag{42}$$

This completes the solution in the Laplace transform domain.

Inversion of the Laplace transforms

To invert the Laplace transform in eqs. (39), (40), and (42), one should adopt a numerical inversion method based on a Fourier series expansion.[30,31] Using this method, the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by

$$f(t) = \frac{e^{ct}}{t_1}\left[\frac{1}{2}\bar{f}(c) + R\,1\sum_{k=1}^{N}\bar{f}\left(c + \frac{i\,k\,\pi}{t_1}\right)\exp\left(\frac{i\,k\,\pi t}{t_1}\right)\right], \quad 0 < t_1 < 2t,$$

where N is a sufficiently large integer representing the number of terms in the truncated Fourier series, chosen such that

$$\exp(ct)R1\left[\bar{f}\left(c + \frac{i\,N\,\pi}{t_1}\right)\exp\left(\frac{i\,N\,\pi\,t}{t_1}\right)\right] \leq \varepsilon_1$$

where ε_1 is a prescribed small positive number that corresponds to the degree of the required accuracy. The parameter "c" is a positive free parameter that must be greater than the real part of all the singularities of $\bar{f}(s)$. The optimal choice of "c" was obtained according to the criteria described in Ref. 30.

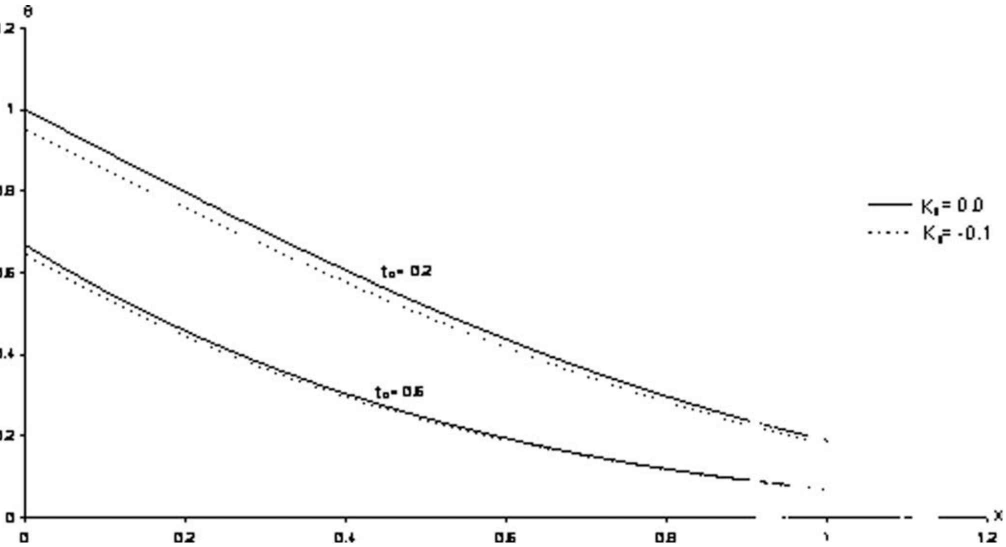

Figure 1 The temperature distribution for different value of K_1.

RESULTS AND CONCLUSIONS

The copper material was chosen for the state of a numerical evaluation, and the constants of the problem were taken as follow:

$$K_o = 386\ \text{N/K.sec}, \quad \alpha_T = 1.78(10)^{-5}\ \text{K}^{-1},$$
$$C_E = 383.1\ m^2/K.sec^2, \quad \eta = 8886.73\ m/sec^2,$$
$$\mu = 3.86(10)^{10}\ \text{N/m}^2, \quad \lambda = 7.76(10)^{10}\ \text{N/m}^2,$$
$$\rho = 8954\ kg/m^3, \quad \mu_o = 4\pi(10)^{-7}\ N.m.sec^2/\ C^2,$$
$$H_o = 10^6\ \text{C/m.sec}, \quad \tau_o = 0.02\ \text{sec}, \quad \varepsilon = 1.60861,$$
$$T_o = 293\ K, \quad a = 0.0104442,$$
$$M = 5.0, \quad K_1 = -0.1.$$

The computations were carried out for $t = 0.4$ s and $\theta_1 = 1$. The temperature, stress, and displacement distributions are represented graphically.

The field quantities, temperature, stress, and displacement depend not only on the state and space variables t and x but also depend on the rise-time parameter t_o and the thermal relaxation-time parameter τ_o. It has been observed that the finite rise-time parameter t_o has significant effect on the temperature, stress, and displacement quantities but. Here, all the variables/parameters are taken in nondimensional forms. Numerical analysis has been carried out by taking x range from 0.0 to 1.0. The numerical values for the field quantities are computed for a wide range of values of finite pulse rise time t_o in the two situations $t > t_o$ and $t < t_o$, respectively.

Figures 1–4 exhibit the space variation of temperature at instants, $t = 0.4$ for different values of t_o in which we observe the following:

(i) Figure 1—significant difference in the value of temperature is noticed for different value of the parameter K_1. Actually, the temperature decreases when the value of the parameter K_1 changes from the normal case ($K_1 = 0$ the thermal conductivity is constant) to $K_1 = -0.1$ ($K_1 \neq 0$ the thermal conductivity is variable) for the two cases, $t > t_o$ or $t < t_o$.

(ii) Figure 2—significant difference in the value of temperature is noticed for different value of the relaxation time τ_o. We noticed that when $t > t_o$ the value of the temperature increases as the value of the relaxation time increases, but the situation is inverse when $t < t_o$ that means the parameter t_o has large effect on the field of the temperature.

(iii) Figure 3—no significant difference in the value of temperature is noticed for different value of the magnetic field when $t > t_o$ or $t < t_o$.

(iv) Figure 4—significant difference in the value of temperature is noticed for the wide rang of different value of the parameter t_o, 0.2, 0.3, 0.4, 0.5, and 0.6.

Figures 5–8 exhibit the space variation of stress at instants, $t = 0.4$ for different values of t_o in which we observe the following:

(i) Figure 5—significant difference in the value of stress is noticed for different value of the parameter K_1. Actually, the absolute value of the stress decreases when the value of the parameter K_1 changes from the normal case ($K_1 = 0$ the thermal conductivity is constant) to $K_1 = -0.1$ ($K_1 \neq 0$ the thermal conductivity is variable) for the case $t < t_o$, but in the other case, $t > t_o$ the value of the absolute value of the stress decreases when the parameter K_1

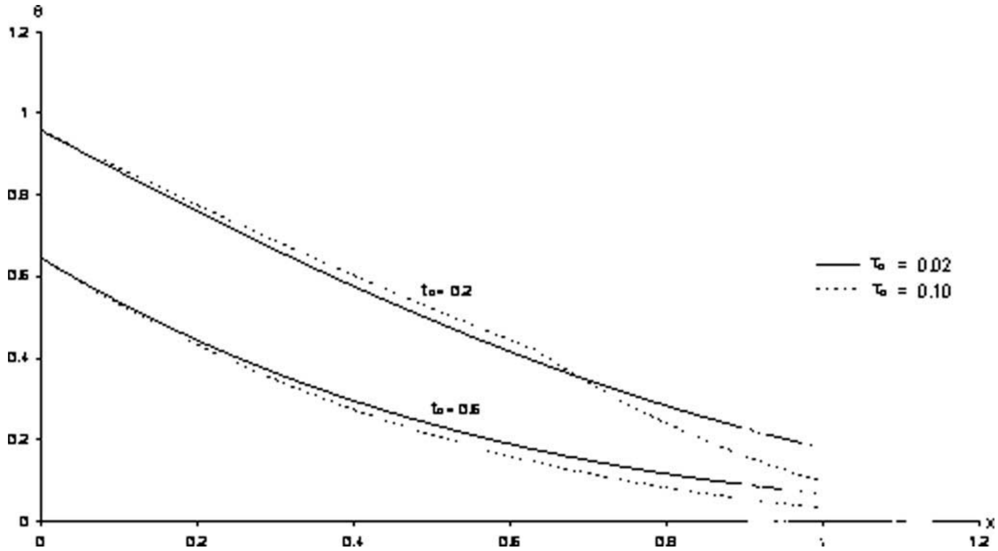

Figure 2 The temperature distribution for different value of relaxation time.

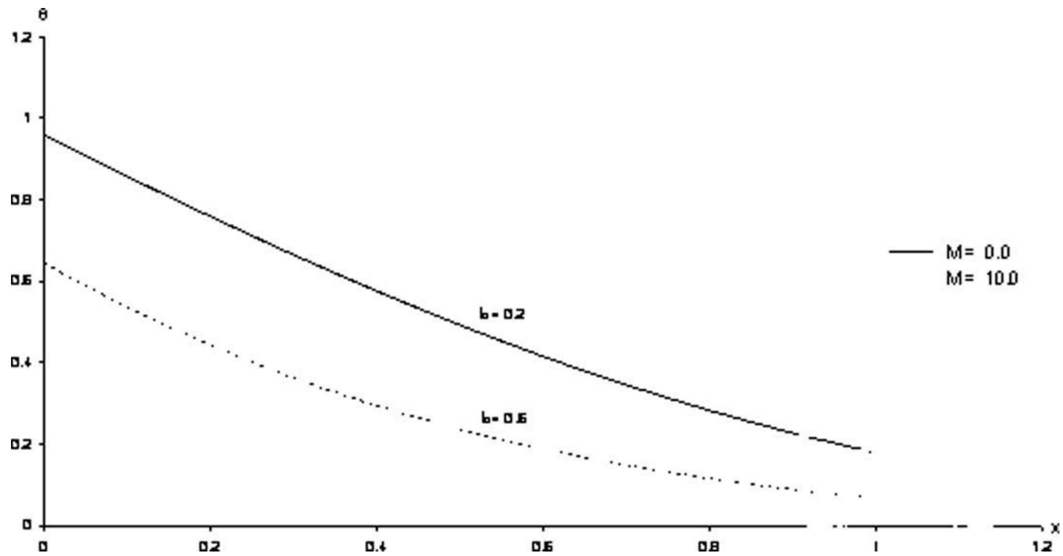

Figure 3 The temperature distribution for different value of magnetic field.

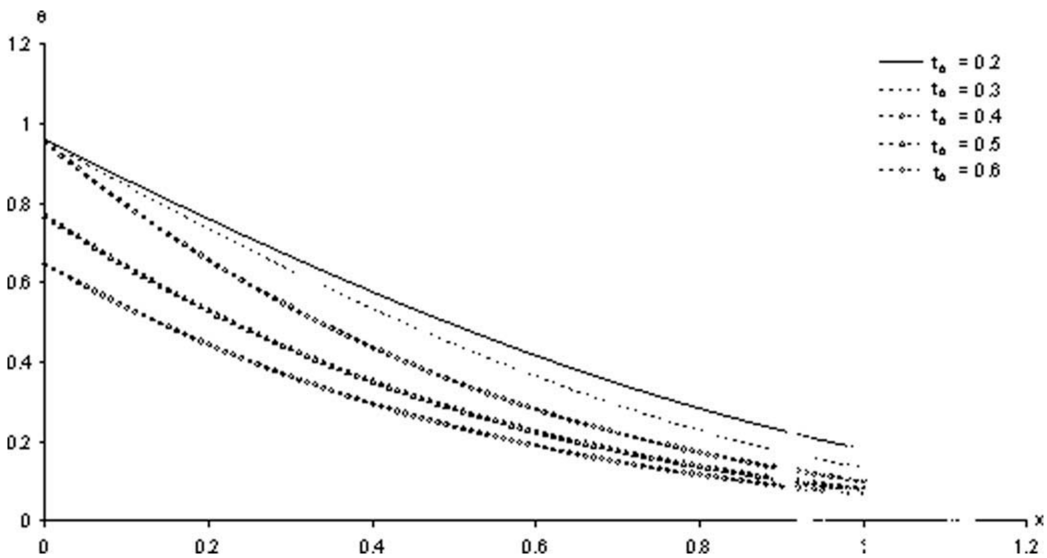

Figure 4 The temperature distribution for different value of t_o.

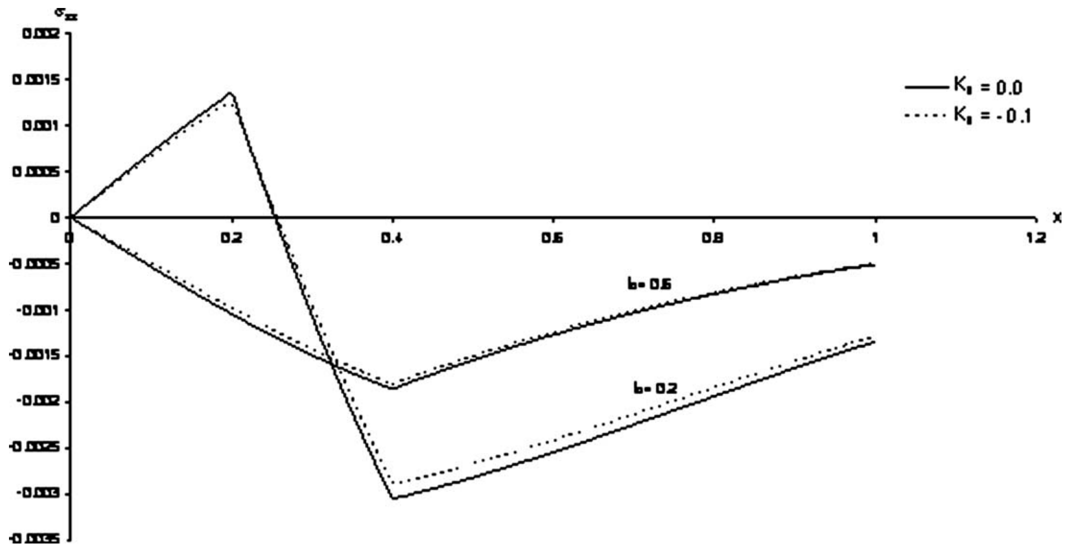

Figure 5 The stress distribution for different value of K_1.

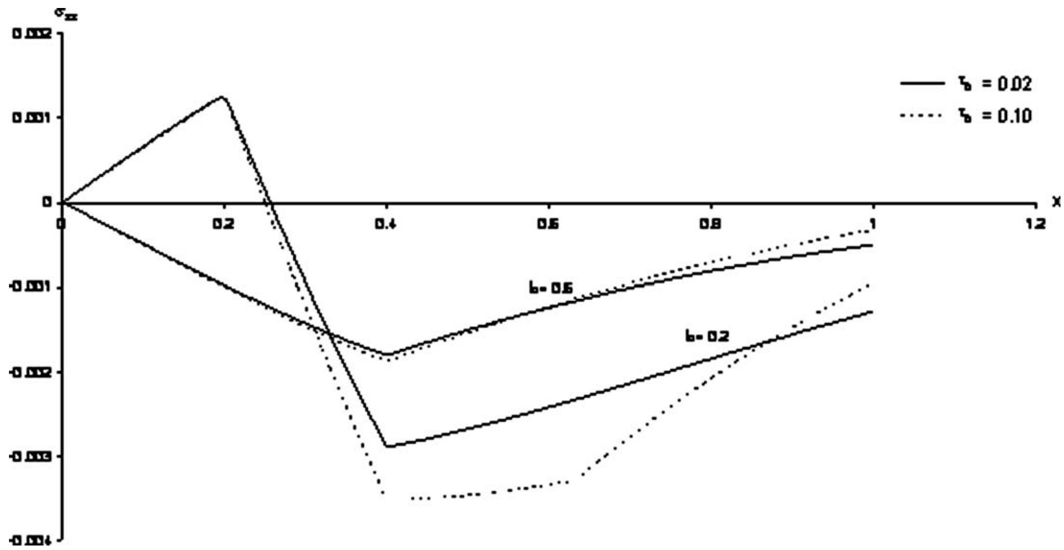

Figure 6 The stress distribution for different value of relaxation time.

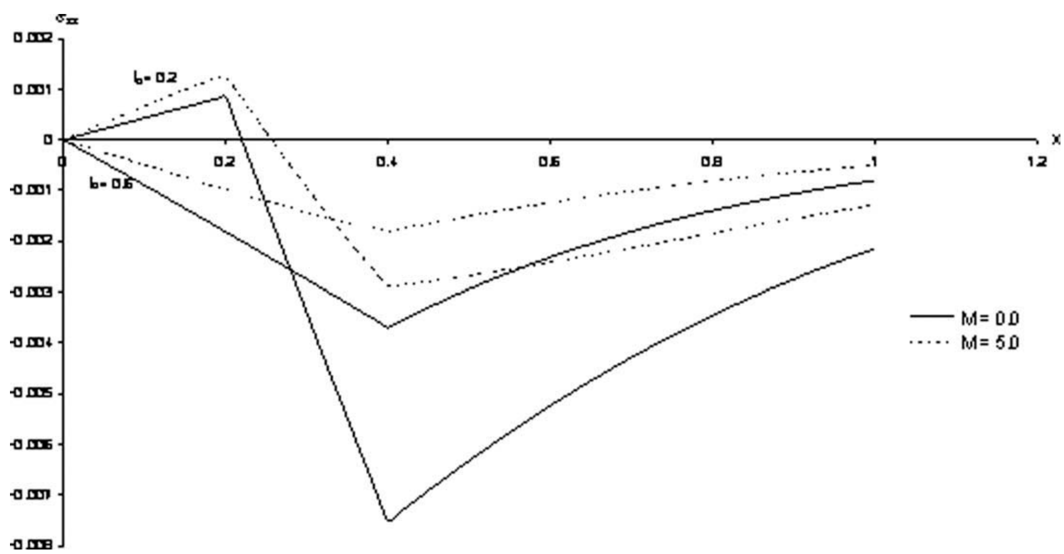

Figure 7 The stress distribution for different value of M.

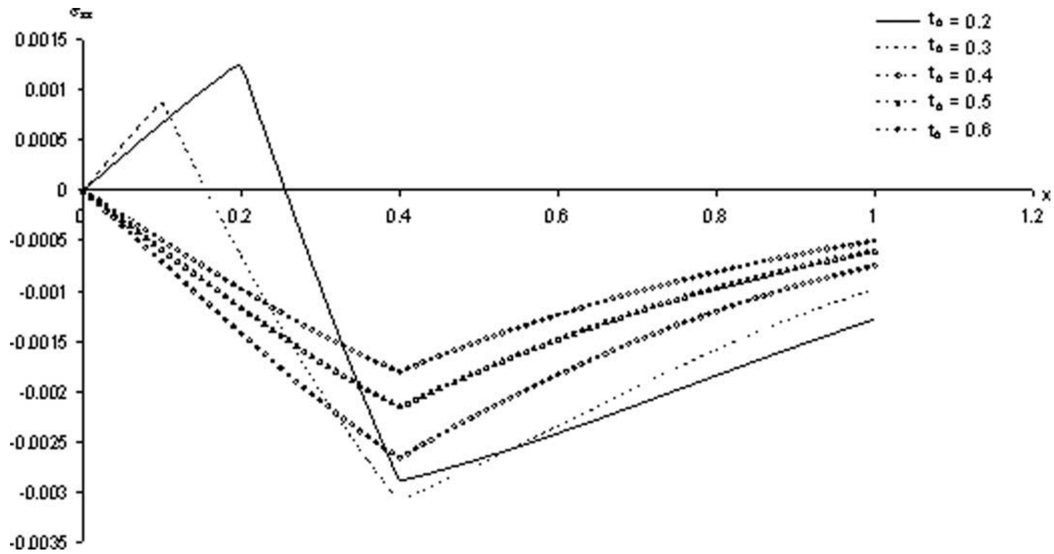

Figure 8 The stress distribution for different value of t_0.

decrease rapidly till $x = 0.2$ and goes down rabidly till $x = 0.38$, after that, it comes up till the end slowly.

(ii) Figure 6—significant difference in the value of stress is noticed for different value of the relaxation time τ_o.

(iii) Figure 7—significant difference in the value of stress is noticed for different value of the magnetic field, where the figure shows that increase in the magnetic field cause increase in the value of the stress at the same point of x for the two cases $t > t_o$ and $t < t_o$.

(iv) Figure 8—significant difference in the value of stress is noticed for the wide rang of different value of the parameter t_o, 0.2, 0.3, 0.4, 0.5, and 0.6.

Figures 9–12 exhibit the space variation of displacement at instants, $t = 0.4$ for different values of t_o in which we observe the following:

(i) Figure 9—small difference in the value of displacement is noticed for different value of the parameter K_1.

(ii) Figure 10—significant difference in the value of displacement is noticed for different value of the relaxation time τ_o. We noticed that when $t > t_o$, the value of the displacement increases as the value of the relaxation time increases, but the situation is inverse when $t < t_o$ for some intervals.

(iii) Figure 11—significant difference in the value of displacement is noticed for different value of the magnetic field, where the figure shows

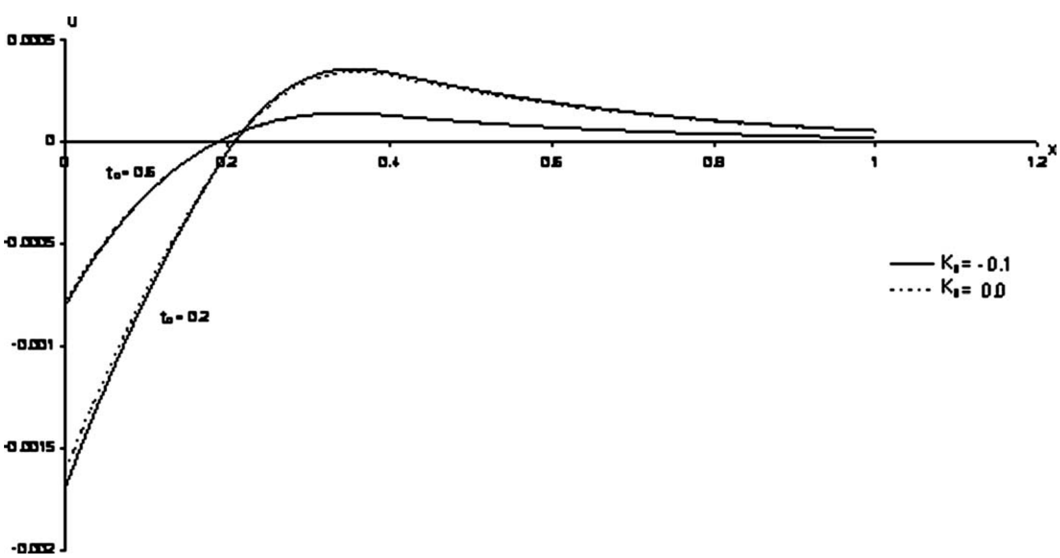

Figure 9 The displacement distribution for different value of K_1.

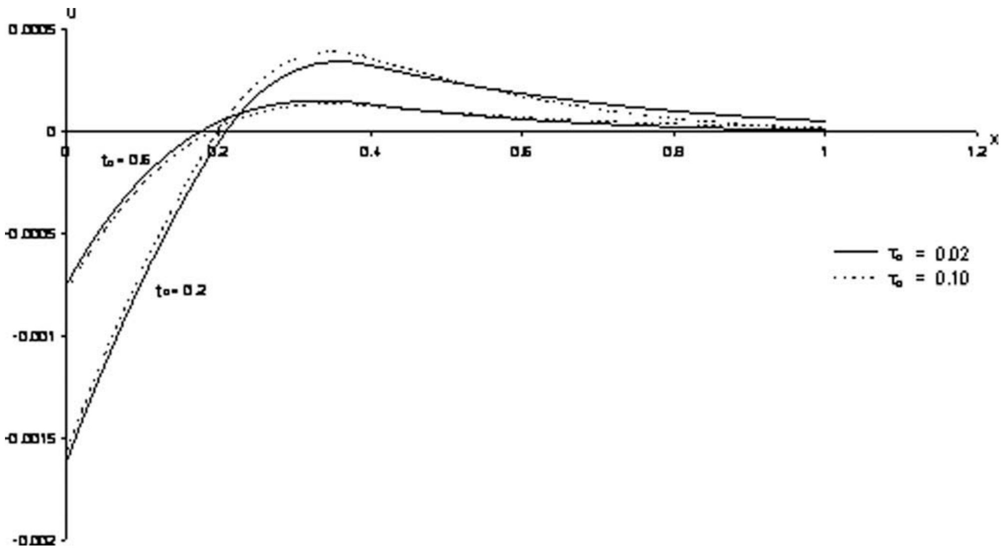

Figure 10 The displacement distribution for different value of relaxation time.

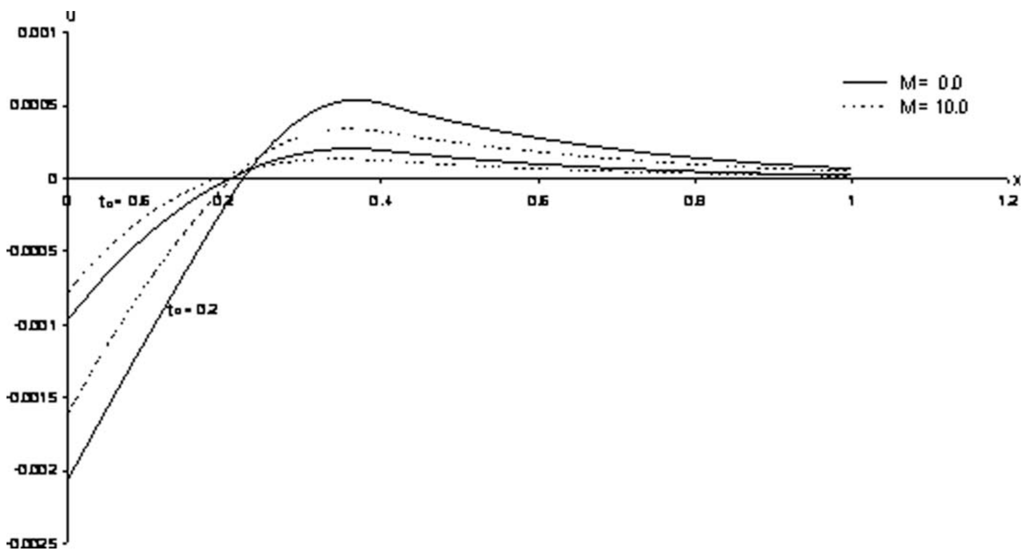

Figure 11 The displacement distribution for different value of *M*.

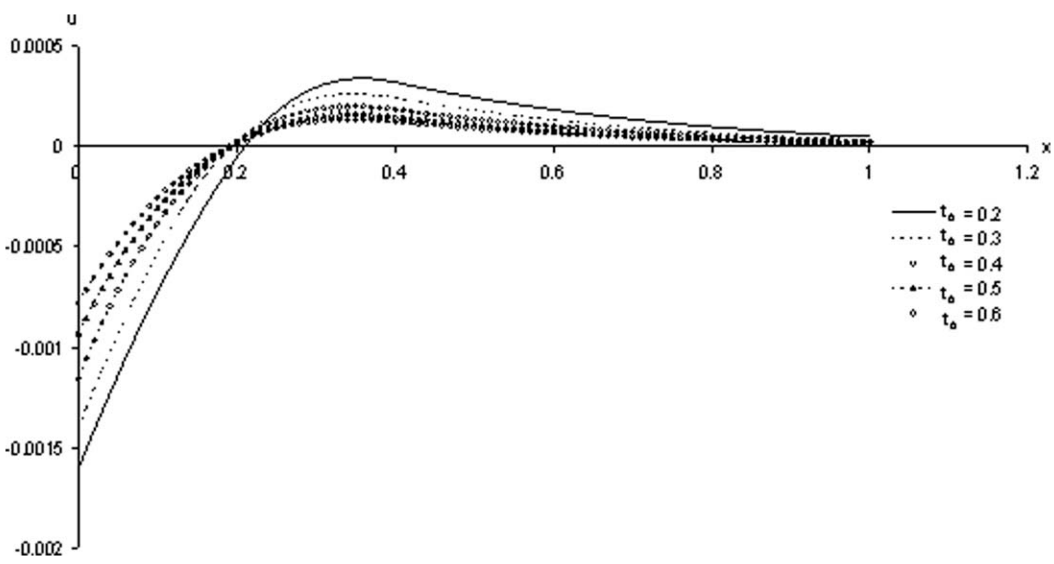

Figure 12 The displacement distribution for different value of t_0.

that increase in the magnetic field cause increase in the value of the displacement at the some points of x for the two cases $t > t_o$ and $t < t_o$.

(iv) Figure 12—significant difference in the value of displacement is noticed for the wide rang of different value of the parameter t_o, 0.2, 0.3, 0.4, 0.5, and 0.6.

Experimental comparison

A comparison will be made between experimental data obtained in Reference 32 and there is some deviation in the shape of curves. This deviation is due to the different of material used in two researches.

NOMENCLATURE

ρ	Density
t	Time
λ,μ	Lamé's constants
T_o	Reference temperature
θ	$= (T - T_o)$ absolute temperature such that $\left\|\frac{\theta}{T_0}\right\| < 1$
u_i	Components of displacement vector
σ_{ij}	Components of stress tensor
e_{ij}	Components of strain deviator tensor
e	$= e_{kk}$, dilatation
δ_{ij}	Delta Kronecker
α_T	Coefficient of linear thermal expansion
γ	$=(3\lambda + 2\mu)\,\alpha_T$
K	Thermal conductivity
K_1	Negative small constant
C_E	Specific heat at constant strain
τ_o	Relaxation time
μ_o	Magnetic permeability
H_o	Magnetic field
E	Electrical field
κ	Diffusivity
L	$= 2.45 \times 10^{-8}$ Lorenz number
σ	Electrical conductivity
c_o^2	$= \frac{\lambda+2\mu}{\rho}$
ε	$= \frac{\gamma}{\rho C_E}$
a	$= \frac{\gamma T_o}{(\lambda+2\mu)(1+k_1 T_o)}$
M	$= \frac{C_E H_o^2 \mu_o^2 \kappa^2 (1+k_1 T_o)}{c_o^2 L T_o}$

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